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DETECTION IN THE PRESENCE OF NONUNIFORM, MIXED SUPPRESSIVE FIRE--ETC(U)

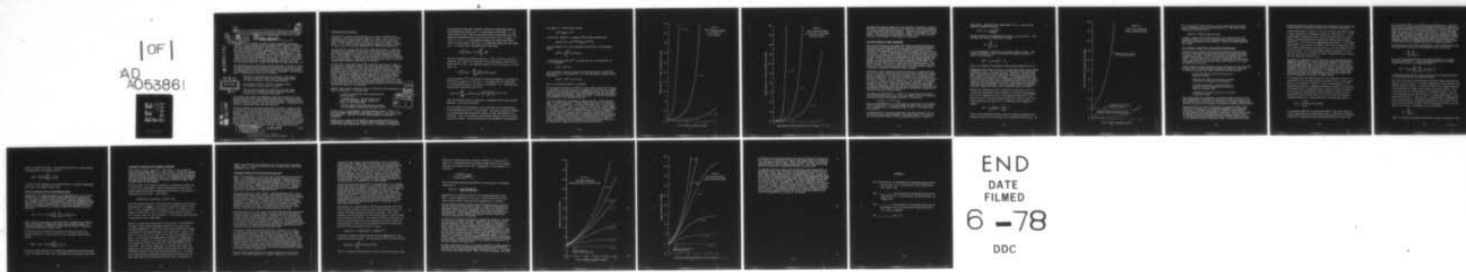
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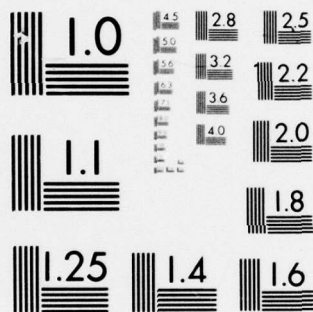
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Detection in the Presence of Nonuniform, Mixed Suppressive Fires,

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Timothy J. Horrigan

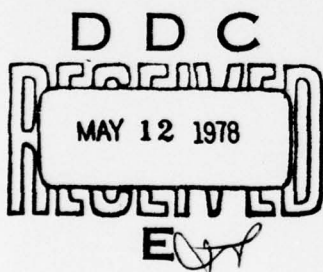
Horrigan Analytics, Chicago, Ill.

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Suppression, as a concept, is given a simple operational explication through a 'single-round period of suppressive effect' which is associated with each projectile impacting in the vicinity of a combatant. During each such single-round period of suppressive effect, which commences at an indicator instant, the affected combatant is *suppressed*; at all other times the combatant is *unsuppressed*. A *period of suppression* for a combatant that is unsuppressed begins with an impact that produces a nonzero, single-round period of suppressive effect; and it ends when the affected combatant first thereafter becomes unsuppressed. Arbitrarily long random periods of suppression for the affected combatant may thus arise from overlap between consecutive single-round periods of suppressive effect.

By proceeding from this definition, expected durations of periods of suppression are deduced under very general conditions for situations in which the impact times of the associated projectiles are adequately represented by independent Poisson processes with constant intensities. The resulting model is mathematically exact, and it includes:

- Arbitrary, random durations for individual single-round periods of suppressive effect that stochastically depend on the miss-distance of the associated projectile
- An arbitrary number of different, nonuniform impact distributions for each type of projectile
- Different distributional characteristics for the single-round period of suppressive effect associated with each distinct pair of projectile-target types



The formulas which result are remarkably simple; they depend only on the average durations of the random single-round periods of suppressive effect and the average arrival rates for the associated rounds. Expected detection times for search processes in which the search activity is suspended during periods of suppression retain the same simplicity.

In those situations the expected durations of a period of suppression and of a period to a detection grow exponentially both with the rates at which projectiles impact and with the average durations of the probabilistically different, single-round periods of suppressive effect. When the detection rate during suppression is small but not identically zero, the corresponding expected detection times can be much smaller than what they are when that rate is identically zero. Indeed, they can become sufficiently small to make all-or-nothing representations of suppressive effect unsatisfactory for many typical applications. Fractional suppression, a more satisfactory concept, is introduced to accommodate nonzero activity rates during suppression.

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SUPPRESSION AT ITS SIMPLEST

Suppression is initially idealized herein as a hiatus introduced into a combatant's activity by the nearby impact of a round. Such a hiatus, when associated with a single round, is defined to start at the time of the impact or other indicator and to continue for a positive duration thereafter. It is termed a *single-round period of suppressive effect*; 'volley' or 'burst' may, of course, be substituted for 'round' when appropriate.

The duration of a single-round period of suppressive effect is inherently voluntary and accordingly may vary widely from combatant to combatant and even from one combatant at a given time to that same combatant at another time. Miss-distance, environment, and round type are additional important sources of variations. However, because the duration is voluntary, speaking of a constant duration is meaningful notwithstanding what may be its actual, probably great variation from instance to instance.

So long as all inter-round impact times exceed the duration of a single-round period of suppressive effect, the total time during which a combatant is suppressed is defined to be the sum of the individual durations. When additional rounds impact during an existing period of suppressive effect, that period will be prolonged, at least until cessation of the single-round period of suppressive effect associated with the last of the additional rounds. A period of suppression for a combatant is consequently defined to terminate when an inter-impact time first exceeds the duration of a single-round period of suppressive effect. The discipline thus prescribed for the idealized combatant is that its combat activities are to be resumed at the expiration of the single-round period of suppressive effect associated with the last impact in its proximity.

Together these concepts determine a nearly irreducibly simple mathematical model of suppression. It requires only

- a region of suppressive affect associated with each combatant
- a constant duration τ for the single-round period of suppressive effect caused by an impact in the affect region
- a Poisson process $N^*(t)$ with constant intensity λ for the impact stream within the affect region

so that λ and τ , two parameters, alone need quantification. $N^*(t)$ is of course the impact point process, the number of impacts in the affect region in a duration t . Define S^* to be the random duration of a period of suppression.

Without loss of generality the combatant may be assumed initially to be suppressed by an impact in its region of suppressive affect at time zero. It will thus remain suppressed at least until τ ; whether it continues to

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be suppressed at some time t depends on whether an appropriate number of timely additional impacts occur. On the hypothesis that $N^*(t) = n$, the impact times of the n rounds in the affect region are uniformly and independently distributed on the interval $[0, t]$ because $N^*(t)$ is Poisson. If T_i^* for $i = 1, 2, \dots, n$ respectively designate the random inter-impact times for those rounds, and if T_{n+1}^* is the duration between the last impact and t , it follows from a theorem¹ of De Finetti that

$$\Pr\left\{\bigwedge_{i=1}^{n+1} T_i^* > t\right\} = \left(1 - \frac{1}{t} \sum_{i=1}^{n+1} t_i\right)_+^n$$

when $(x)_+$ designates the positive part of x : $(x)_+ = 0$ for $x < 0$, and $(x)_+ = x$ otherwise. By virtue of an identity² for the realization of none out of m events (with $m = n+1$), the probability that all the T_i^* are equal to or less than τ is:

$$\Pr\left\{\bigwedge_{i=1}^{n+1} T_i^* \leq \tau\right\} = \sum_{k=0}^{n+1} \binom{n+1}{k} (-1)^k (1 - k\tau/t)_+^n.$$

Since the duration S^* of the period of suppression exceeds t if and only if all the T_i^* are equal to or less than τ , it follows that the right member of the preceding equation is in fact $\Pr\{S^* > t | N^*(t) = n\}$. Therefore, the unconditional probability that $S^* > t$ is

$$\Pr\{S^* > t\} = \sum_{k=0}^{\infty} (-1)^k [\lambda(t - k\tau)_+]^{k-1} \left[\frac{\lambda(t - k\tau)_+ + k}{k} \right] e^{-\lambda[t - (t - k\tau)_+]}$$

after the resultant order of summations is exchanged and the inner extended summation is put into closed form.

The right member of this equation is not convenient for the determination of the expected value of S^* or its variance. Its Laplace transform, however, is both convenient and intrinsically useful, as later considerations will illustrate. Let \mathcal{L} be the Laplace transformation operator, and let s be the transform variable. Termwise application of the fundamental transformation

for powers of t , which may be written

$$\mathcal{L}(t^{m-1}/(m-1)) = s^{-m}$$

for positive, integral m , together with the shift theorem yields

$$\mathcal{L}[\Pr\{S^* > t\}] = [1 - e^{-(\lambda+s)\tau}] / [s + \lambda e^{-(\lambda+s)\tau}] .$$

Since the moments of S^* can be obtained from $\Pr\{S^* > t\}$ in the following manner

$$E(S^{*n}) = n \int_0^\infty t^{n-1} \Pr\{S^* > t\} dt ,$$

it follows directly that $E(S^*)$ is merely the value of $\mathcal{L}[\Pr\{S^* > t\}]$ at $s = 0$; therefore,

$$E(S^*) = (e^{\lambda\tau} - 1) / \lambda .$$

The second moment similarly follows from the derivative of $\mathcal{L}[\Pr\{S^* > t\}]$ with respect to s as evaluated at $s = 0$; and the variance clearly follows therefrom as

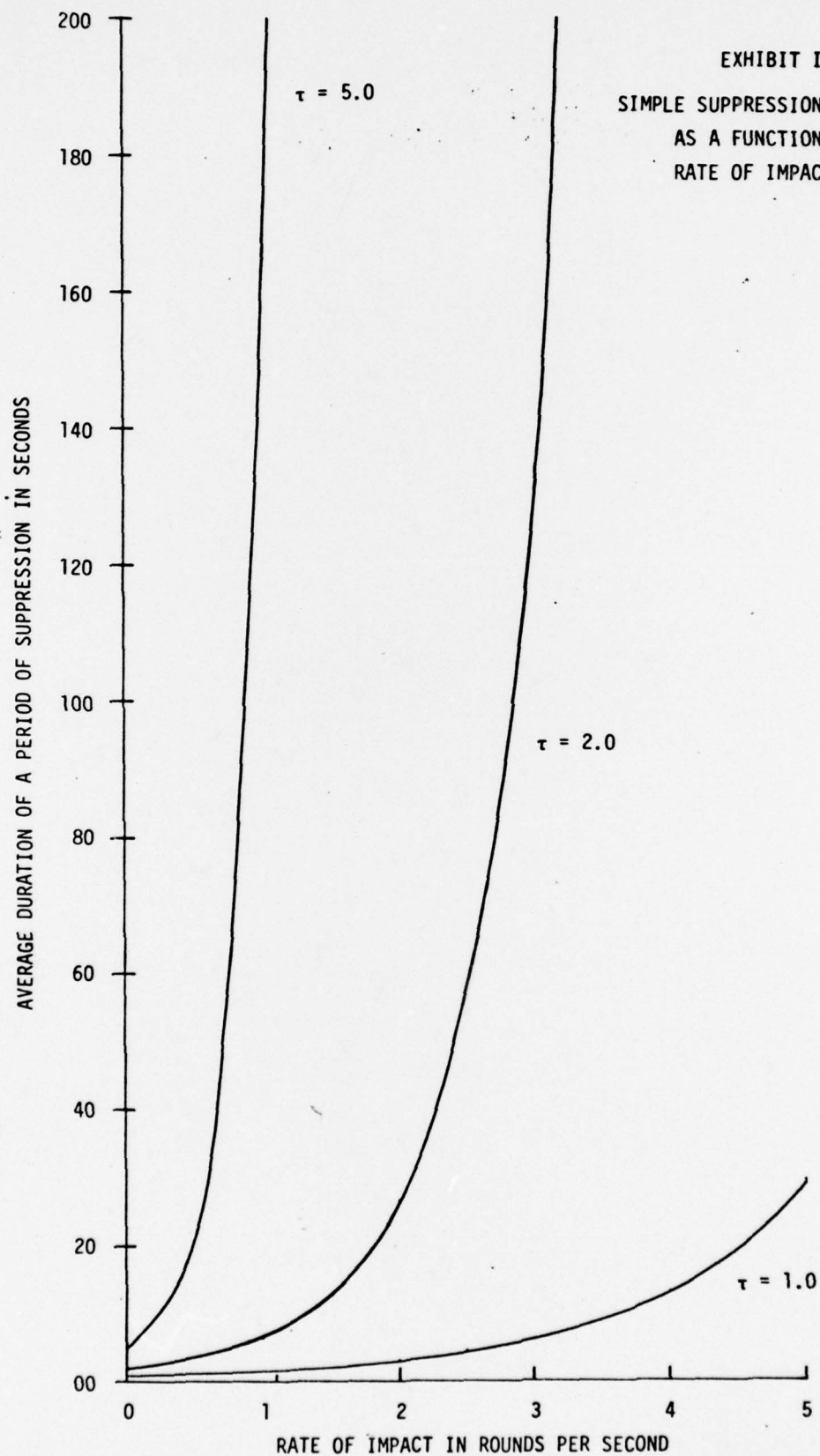
$$\text{Var}(S^*) = (e^{2\lambda\tau} - 2\lambda\tau e^{\lambda\tau} - 1) / \lambda^2 ,$$

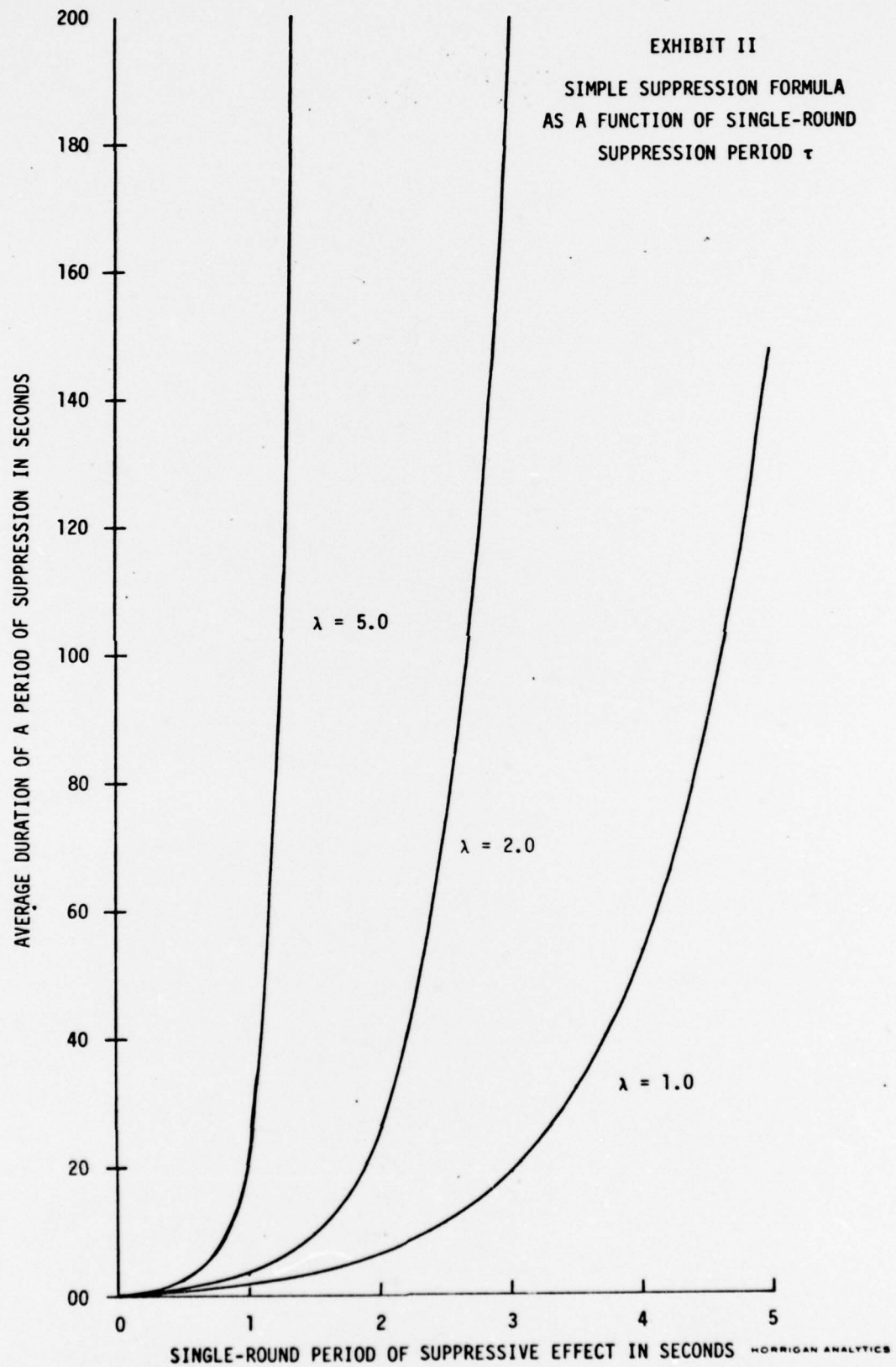
after the appropriate algebra is performed.

The exponential dependency of $E(S^*)$ on λ and τ implies that small increases in the impact rate in the course of an engagement can induce large, sudden increases in the average duration of suppression periods, once a moderate impact rate has been achieved. The similar growth in the variance suggests very substantial fluctuations in those durations. In fact the coefficient of variation for S^* is asymptotically one.

Just how rapidly $E(S^*)$ can change is shown by Exhibit I, following this page. For selected durations τ of the single-round period of suppressive effect, $E(S^*)$ is graphed as a function of the impact rate λ in the region of suppressive affect. When τ is as small as two seconds, slight changes in the impact rate can produce great changes in $E(S^*)$, the average duration of a period of suppression. As Exhibit II shows, those great changes in the average duration of suppression in response to slight variations in the impact rate are matched by the correspondingly great changes caused by slight variations in the duration of a single-round period of suppressive effect. Consequently small discrepancies between assumed durations of suppressive effect and actual durations can introduce great variations in any durations of suppression periods extrapolated therefrom.

EXHIBIT I
SIMPLE SUPPRESSION FORMULA
AS A FUNCTION OF
RATE OF IMPACT λ





Although these formulas appear new in the context of suppression, they are well-known in other applications; in fact they have a surprising propensity for being rediscovered in new contexts³. They may also be derived by more general means than those herein employed, notably by the methods of renewal theory. The derivation just outlined is, however, direct and is the one that led to the formulas in the context of suppression.

DETECTION IMPEDED BY SIMPLE SUPPRESSION

Many search activities in the combat environment are characterized by an exponentially distributed detection time. Any such activity consequently possesses the Markov property for the exponential distribution⁴ and is therefore easily adjusted to account for being suspended during periods of suppression. Indeed, a detection may occur only between periods of suppression because the hiatuses they create block all such events while they last; in other respects the search and bombardment activities are presumed independent. The Markov property then insures that the random detection time retains the *same* exponential distribution regardless of the number and duration of preceding periods of suppression and fruitless search. Since $N^*(t)$ is Poisson, it similarly insures that the duration between the end of one period of suppression and the start of the next defines a family of independent, identically distributed random variables.

Accordingly a basic suppression-search cycle exists. It begins with the onset of a period of suppression and ends either with the onset of another period of suppression or a detection, whichever first follows the initial period of suppression. All cycles are identically and independently distributed in duration. The first part of a cycle of course has the duration S^* , that of a simple period of suppression. The last part is the period between the cessation of suppression and either a detection or an impact, whichever occurs first. Since the search activity and the bombardment activity are independent aside from periods of suppression, the probability distribution for the duration from the end of the period of suppression to the end of the cycle follows directly.

Designate that duration by T^* . Since T^* is the minimum of the time to the first detection and the time to the next impact, which are independent, exponentially distributed random variables, it follows that

$$\Pr\{T^* > t\} = e^{-(\lambda+\gamma)t}, \quad t \geq 0$$

when γ is the detection rate in the absence of suppression. A cycle thus has the duration $S^* + T^*$; and the probability that it ends with a detection, an event which is independent of both S^* and T^* , is easily shown to be $\gamma/(\gamma+\lambda)$.

A combatant that is initially suppressed at the time zero may or may not end its first cycle with a detection. The random number of cycles up to and including that on which its first detection occurs has a geometric

distribution. Designate that random number by N^* ; it then has the geometric probability density

$$\Pr\{N^* = n\} = \frac{\gamma}{\gamma + \lambda} \left[\frac{\gamma}{\gamma + \lambda} \right]^{n-1}.$$

Further, designate the random duration of the i -th cycle by C_i^* . Then the time D^* to the first detection is simply

$$D^* = \sum_{n=1}^{N^*} C_n^*$$

a sum of independent, identically distributed random variables. The average time $E(D^*)$ to the first detection following the onset of a period of suppression is

$$E(D^*) = (1/\lambda + 1/\gamma)e^{\lambda\tau} - 1/\lambda,$$

which is an immediate consequence of the preceding expression for D^* .

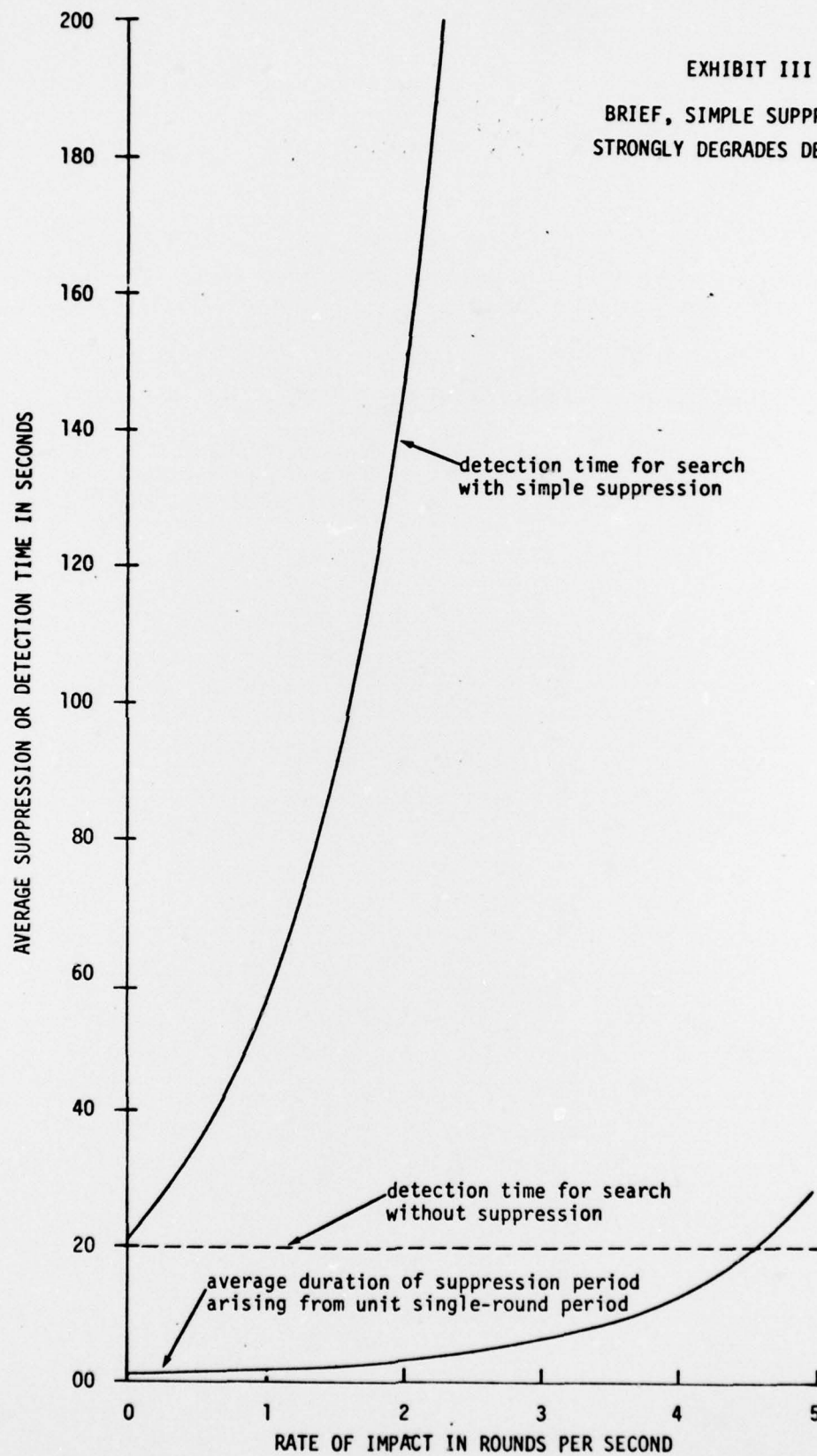
Suppression, when taken as a hiatus, is thus seen to have a great effect on detection times. They grow at a rate even greater than the simple suppression periods previously examined. Exhibit III, following this page, illustrates that rapid growth when the average detection time in the absence of suppression is 20 seconds. The average detection time in the presence of suppression is displayed as a function of the impact rate for a single-round period of suppressive effect of unit duration in comparison with the average duration of a single period of suppression under the same circumstances. The strong effect that all-or-nothing periods of suppressive effect have on detection times is manifest.

Because the random duration of a suppression-search cycle is $S^* + T^*$, a sum of two independent random variables, the Laplace transformation $C(\cdot)$ of its frequency function is the product of those for S^* and T^* . Since that for S^* follows directly from that of its tail, which is already established, and that for T^* is immediate, their respective product

$$C(s) = \frac{\lambda + s}{\lambda + se^{(\lambda + s)\tau}} \cdot \frac{\lambda + \gamma}{\lambda + \gamma + s},$$

with s as the transform variable, gives the Laplace transform of the frequency function for the duration of a suppression-search cycle. As

EXHIBIT III
BRIEF, SIMPLE SUPPRESSION
STRONGLY DEGRADES DETECTION



D^* is the sum of N^* such variables, which are identically distributed and are independent both of each other and of N^* , the Laplace transformation of the tail of D^* is

$$\mathcal{L}\Pr\{D^* > t\} = \frac{\lambda + \gamma}{s} [1 - C(s)] / [\lambda + \gamma - \lambda C(s)]$$

in which s remains the transform variable. The expected value of D^* , already derived, as well as the higher moments can of course be easily and directly obtained from this equation. Later a more significant use will emerge.

MISS-DISTANCES, WEAPON MIXES, AND GENERALIZED SUPPRESSION

No doubt the most apparent unsatisfactory assumption underlying these formulas is the idealized constant duration of the single-round period of suppressive effect. Further, that duration is required to be independent of miss-distance, and it must be the same for each type of round. Ignoring casualties is of course a shortcoming, but the suppression process itself is not thereby grossly restricted, as it is by the aforementioned assumptions.

Several avenues of generalization for the simple model are thus suggested; and they lead to broadly applicable formulas of remarkable simplicity. The generalized suppression model established therefrom permits:

- Random durations for single-round periods of suppressive effect
- Durations for single-round periods of suppressive effect that depend on miss-distance
- Distinct characteristics for the periods of suppressive effect associated with each ordnance or projectile type
- Segregated, nonuniform delivery of any mixture of projectile types

The general model thus encompasses a substantial number of factors that affect suppression. Durations of suppression for each round type are not only permitted to be distinct, but also they may be random variables with different probability distributions, which may be functions of miss-distance.

Random durations for single-round periods of suppressive effect allow differences in judgment of an individual combatant to be reflected as variations in the single-round suppressive effect of even identical rounds impacting at the same distance. Durations of single-round periods of suppressive effect that deterministically depend on miss-distance are thereby randomized regardless and thus illustrate another variation in the suppressive effect of identical rounds. Permitting single-round

periods of suppressive effect to depend on miss-distance also allows local nonuniformities in projectile delivery to be faithfully represented.

In the simple model all impacts in the area of suppressive affect produce a single-round period of suppressive effect of fixed duration τ ; in the general model a projectile of the i -th type fired from the j -th source produces a single-round period of suppressive effect with the random duration T_{ij}^* , all of which are independently distributed. In the simple model there is only one rate for impact in the area of suppressive affect; in the general model there is one such rate λ_{ij} for each projectile type from each source. The respective impact times of projectiles of each type from each source are assumed to follow independent Poisson processes with the respective intensities λ_{ij} .

Presumably the duration of a single-round period of suppressive effect depends on miss-distance. Given a particular combatant and situation, a particular projectile type, and a fixed miss-distance x , there is a random variable $T^*(x)$ which is the duration of the single-round period of suppressive effect that results from an impact a distance x from the combatant. Of course, the duration of such a suppression period may be taken as function of the miss-distance. In either event, because the miss-distance itself is a random variable, the resulting single-round period of suppressive effect has a random duration.

As indicated above the random duration of this suppression period for a projectile of the i -th type from the j -th source is T_{ij}^* in which dependency on miss-distance is implicit. If the function $s_i(t, x)$ is the probability density for a single-round period of duration t arising from the impact of the i -th projectile type a distance x from the combatant, and if $f_{ij}(x)$ is the probability density governing impacts at x by a projectile of the i -th type from the j -th source, then the expected (average) duration of a single-round period of suppressive effect is

$$E(T_{ij}^*) = \int_0^\infty \int_0^\infty t s_i(t, x) f_{ij}(x) dx dt.$$

The remarkable aspect of the generalized model is that these expected values together with the average impact rates λ_{ij} determine the expected duration of a suppression period and expected detection times as well.

As in the simple model, for an entity to be suppressed for a duration t , there must be an unbroken chain of overlapped, single-round suppression periods which together, from the beginning of the first to begin, to the end of the last to cease, constitute a duration t . Unlike the simple model, the durations of the single-round periods of suppressive effect are no longer the same in duration; short ones and long ones are haphazardly mixed, and many gaps between short ones may be filled by a single long one. Despite this great increase in physical complexity and a comparable increase in mathematical difficulty, there is little change in the formula for the expected duration of a suppression period.

For R round types and N fire sources define λ , the combined impact rate of projectiles in the region of suppressive affect, as follows:

$$\lambda = \sum_{i=1}^R \sum_{j=1}^N \lambda_{ij}.$$

As in the simple model, designate the random duration of an overall suppression period by S^* . Then the expected duration of an overall suppression period in the generalized model is

$$E(S^*) = \frac{1}{\lambda} \left\{ \exp \left[\sum_{i=1}^R \sum_{j=1}^N \lambda_{ij} E(\tau_{ij}^*) \right] - 1 \right\},$$

a remarkably simple formula, which involves only the expected durations of single-round periods of suppressive effect.

When each round type is represented by a distinct single-round period of suppressive effect which is a constant independent of miss-distance, the formula simplifies further. In that case there are no random variations in the duration of a single-round period of suppressive effect. For a fixed round type all such periods are of identical duration. For the i -th round type designate the duration of a single-round period of suppressive effect by τ_i . Because the τ_i are functionally independent of miss-distance, they are consequently independent of the source of fire. Hence, the segregation of impact rates by the source of fire is not necessary in this case. Accordingly, if λ_i is defined by

$$\lambda_i = \sum_{j=1}^N \lambda_{ij},$$

then it designates the impact rate of the i -th type of projectile in the

region of suppressive affect. The expected duration of an overall period of suppression is accordingly given by

$$E(S^*) = \frac{1}{\lambda} \left\{ \exp \left[\sum_{i=1}^R \lambda_i \tau_i \right] - 1 \right\}$$

in which S^* again designates the random duration of an overall suppression period and λ the combined impact rate.

EXPECTED DETECTION TIMES IN THE GENERALIZED MODEL

Detection in the generalized model is conceptualized just as it is in the simple model. A combatant cycles between suppression and search until it first makes a detection before the onset of the next suppression period. Despite the greatly increased physical complexity encompassed by the general model there is no proportionate increase in the complexity of the formula for expected detection times. With D^* again designating the random time to a detection by an initially suppressed combatant, it can be shown that

$$E(D^*) = (1/\gamma + 1/\lambda) \exp \left[\sum_{i=1}^R \sum_{j=1}^N \lambda_{ij} E(T_{ij}^*) \right] - 1/\lambda$$

when γ remains the parameter in the exponential distribution of detection time in the absence of suppressive fires. Thus a simple, general, and convenient formula is available for connecting the effect of suppressive fires with the ability to return fires.

When the durations of single-round periods of suppressive effect are assumed constant for a given projectile type a somewhat simpler formula governs:

$$E(D^*) = (1/\gamma + 1/\lambda) \exp \left[\sum_{i=1}^R \lambda_i \tau_i \right] - 1/\lambda$$

in which τ_i again represents the single-round suppression duration assigned to the i -th projectile type, and λ_i designates the corresponding impact rate.

DURATIONS OF SUPPRESSION FOR UNDAMAGED COMBATANTS

These formulas neglect causalities. While that is a minor omission relative to the simple model, it is still a flaw. It tends to lengthen expected detection times because it implicitly ignores the fact that an entity must survive in order to detect. The condition that a combatant survives the rounds impacting in its area of suppressive affect during the suppression periods preceding its making a detection reduces the expected number of such impacts.

How the duration of a period of suppression is affected is easily seen in terms of the simple model. With δ designating the single-round damage probability and $u(t)$ designating the event that the combatant is undamaged during the time t , the formula

$$E[S^* | u(S^*)] = [(1 + \delta\lambda\tau) / (\delta + 1 - \delta e^{-\lambda\tau}) - 1] / \lambda$$

gives the expected duration of those periods of suppression during which the combatant is undamaged. In situations in which no damage is possible δ is zero, and $E[S^* | u(S^*)]$ then equals $E(S^*)$. For positive δ it is always less than $E(S^*)$; and it strictly decreases with increasing δ until finally, when δ is one, it becomes τ , the smallest possible period of suppression in the simple model.

Whether the quantitative consequences of using $E(S^*)$ vice $E[S^* | u(S^*)]$ are major or minor obviously depends strongly on the single-round casualty probability δ . When it is small and the impact rate is small to moderate, the consequences appear to be negligible. However, whenever it is not small or the impact rate is high, the consequences are major. In such cases the consequences are greater for damaged combatants; for instance, if δ is small and λ moderate, then $E[S^* | u(S^*)]$ can be about ten percent less than $E(S^*)$, while $E[S^* | \bar{u}(S^*)]$ can be twice $E(S^*)$. On the other hand, when δ is moderate and λ high, the reverse can easily obtain; $E[S^* | u(S^*)]$ can be about half $E(S^*)$, while $E[S^* | \bar{u}(S^*)]$ exceeds it by no more than ten percent or so. In either case, those periods of suppression during which casualties occur are much longer than those during which there are none. Combatants, in

effect, are pinned down by suppressive fires for much longer times when damage occurs -- a possibly surprising fact considering the assumed total randomness of the fires.

FRACTIONAL SUPPRESSION AND EXPECTED DETECTION TIMES

Neglect of casualties is not the only flaw in the generalized suppression model. A more fundamental one is the idealization of suppression as a hiatus in the activity of the suppressed combatant. Although that handy idealization is commonly used in modeling suppression, it is none the less counterfactual. Suppressive fires slow down activities; they do not necessarily stop them. Idealizing suppression as a hiatus is adequate only insofar as periods of suppression are considered in abstraction -- without any interaction with combat activities.

Search activities are a case in point. Expected detection times in the presence of suppressive fires can very easily become very long, as Exhibit III illustrates. Simply because those times can be so long, the difference between suppression as a stopping of all activity and suppression as a slowing of it is important. If suppression is truly a hiatus in combat activities, then detections cannot be made during periods of suppression, regardless of their durations. If suppression is anything less total, however, detections will then frequently be made during periods of suppression, particularly when their expected durations are long.

Suppression that is less than total is herein termed *fractional suppression*; during periods of fractional suppression combat activities proceed at a fraction of their unsuppressed rates. Search activities of the type previously defined, that proceed with a search rate γ in the absence of suppression, proceed with the reduced, fractional rate $\eta\gamma$ (for an appropriate η in the unit interval) during periods of suppression. Expected detection times therefore can never exceed $1/(\eta\gamma)$ regardless of the duration of periods of suppression. Fractional suppression and casualty production thus both operate to decrease the duration of detection times.

Idealizing a single-round period of suppressive effect not as a hiatus in a search activity but as reduction in some major factor, for example the solid angle available to the combatant for search, captures a vital characteristic of the interaction of search and suppression. A limit on the efficacy of suppressive fires to inhibit detection is imposed; a point of diminishing return is established. Increasing rates of fire no longer produces progressively greater increases in expected detection times. Instead, successive increases reach a maximum and then become progressively smaller; and the expected detection time can never be forced beyond $1/(\eta\gamma)$. A necessary logical boundary is thus incorporated without which the suppression process itself is compromised.

What fractional suppression means is easily visualized in terms of the example. An upright combatant, for example, typically has a field of

view that is much greater than that available from a crouching or a prone position. Nearby impacts which result in that combatant's taking temporarily a crouching or a prone position thereby introduce fractional suppression by reducing the solid angle available for search activity from that available in an upright position to some smaller portion. As a result the search rate is decreased, and the expected detection time is increased. Impacts in the vicinity of a crouching combatant similarly can cause the solid angle available for search activities to be reduced to that portion available from a prone position. Thus conceived, fractional suppression makes the counterfactuality of suppression as a hiatus obvious.

Quantifying fractional suppression is straightforward. The fraction n itself, in terms of the example, is merely the ratio of the steradian of the solid angle available to the search activity in the presence of suppression to that available in the absence of suppression. The search activity can accordingly be represented by two independent processes, one characterized by the search rate $(1-n)\gamma$ and the other by the search rate $n\gamma$. The first process arises from search in the solid angle that is unavailable during periods of suppression; the second process arises from search in the solid angle that is always available. Suppression always suspends the first process, but it never affects the second.

Consequently, the random detection time D_1^* associated with the first process behaves exactly the same as the random detection time in the presence of simple suppression previously examined. That random detection time D_2^* associated with the second process of course follows the exponential distribution. The random time $D^*(n)$ at which the combatant, cycling between fractional suppression and search, makes its next detection is clearly just the minimum of those two random times. The tail of the distribution of $D^*(n)$ is thus

$$\Pr\{D^*(n) > t\} = \Pr\{D_1^* > t, D_2^* > t\} = \Pr\{D_1^* > t\}e^{-n\gamma t},$$

in which the right-most member follows from the independence of the underlying search processes. The n -th moment of $D^*(n)$ is thus given by

$$E[D^*(n)]^n = n \int_0^\infty t^{n-1} \Pr\{D_1^* > t\} e^{-n\gamma t} dt,$$

which is essentially nothing other than the $(n-1)$ -th derivative of the

previously established Laplace transform $\mathcal{L}\Pr\{D^* > t\}$ of the tail of D^* , after $\eta\gamma$ is substituted for the transform variable and $(1-\eta)\gamma$ is substituted for the original search rate. Consequently, if the variable z is defined by

$$z = \frac{(\lambda + \eta\gamma)[\lambda + (1-\eta)\gamma]}{(\lambda + \gamma)[\lambda + \eta\gamma e^{(\lambda + \eta\gamma)\tau}]},$$

then the expected detection time $E[D^*(\eta)]$ in the presence of fractional suppression is

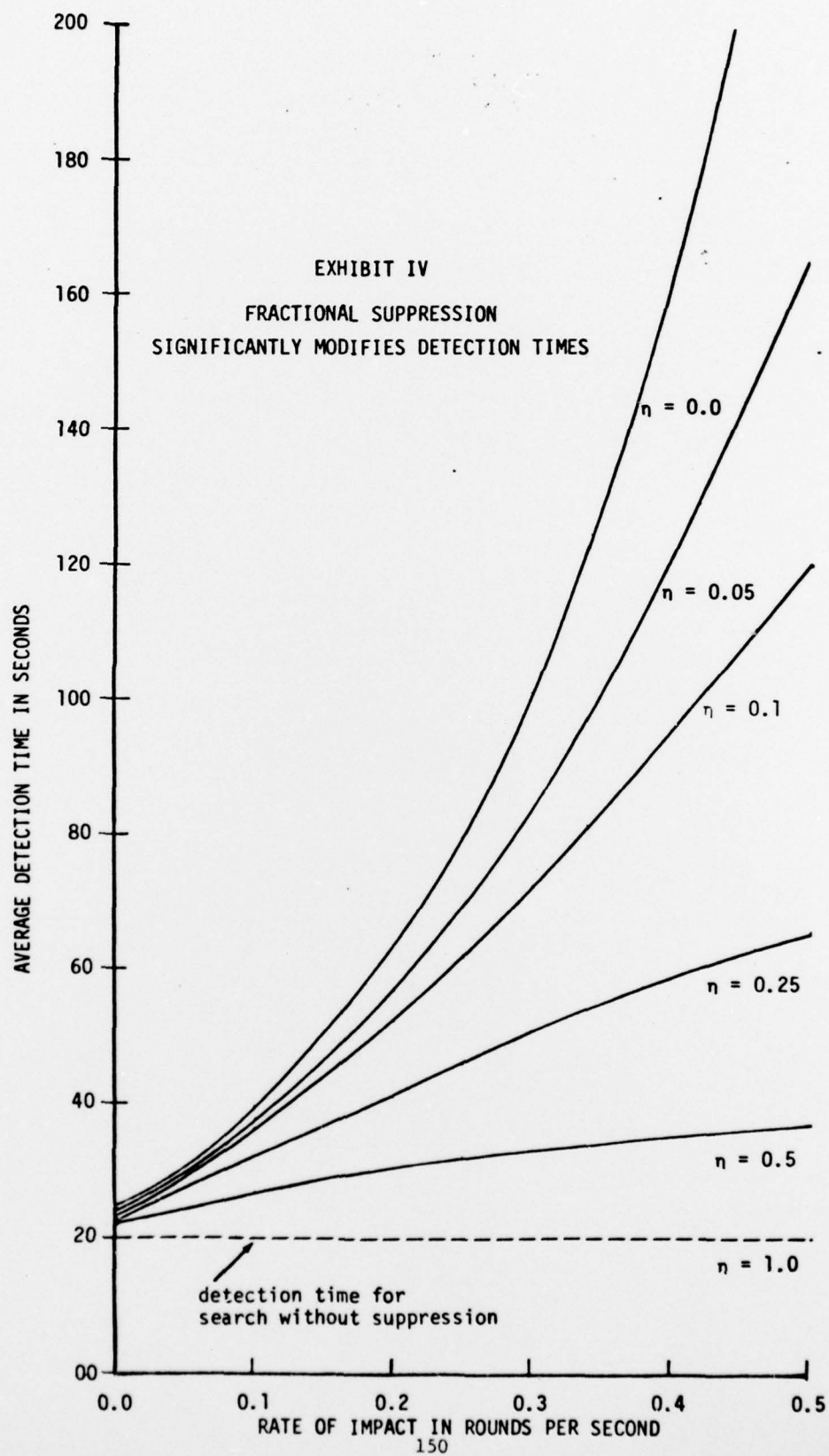
$$E[D^*(\eta)] = \frac{[\lambda + (1-\eta)\gamma](1-z)}{\eta\gamma[(1-\eta)\gamma + \lambda(1-z)]}.$$

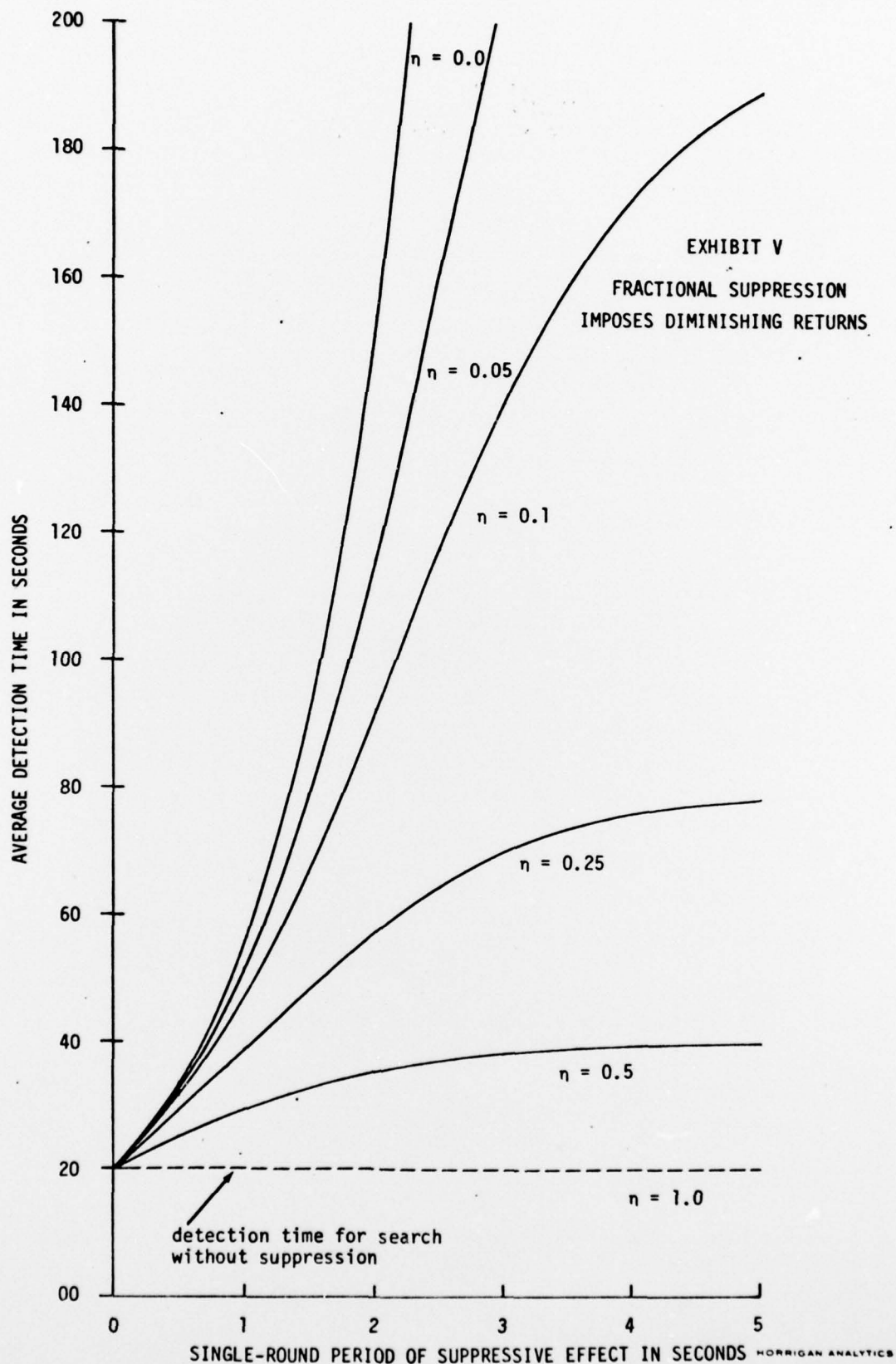
Regrettably, the algebraic simplicity of the expected detection time $E(D^*)$ in simple suppression is lost, but a vital recognition of diminishing returns, which is much more than compensatory, is acquired.

How fractional suppression affects expected detection times is shown in Exhibits IV and V, which follow this page. In both those exhibits the suppression fraction η takes the values: 0.0, 0.05, 0.1, 0.25, and 0.5. The first value, of course, corresponds to the usual idealization of suppression as a hiatus; the values from 0.05 to 0.25 are perhaps more representative. In each exhibit the expected detection time in the absence of suppression is 20 seconds.

In Exhibit IV the single-round period of suppressive effect is 5 seconds, and the rates of impact λ are small to moderate, yet variations in the expected detection times are great. When λ is about 0.1 the range is already significant, and it increases substantially with increases in λ . When λ equals 0.5 the slight difference in the suppression fraction η between total suppression ($\eta = 0$) and nearly total suppression ($\eta = 0.05$) results in an almost 40 percent reduction in the expected detection time. The difference in detection times arising from total suppression and the next level of reduced activity ($\eta = 0.1$) exceeds 50 percent. If η is about 0.1 instead of 0, then the expected detection time is overestimated by 120 percent. The percentage differences increase slightly with smaller expected detection times for detection in the absence of suppression and decrease slightly with larger ones.

A single, high impact rate ($\lambda = 1$) is used in Exhibit V, and the expected detection times for the selected suppression fractions are graphed as functions of the single-round period of suppressive effect τ . The effect of the high impact rate is plain. When τ is about 2.5 seconds, the range





of detection time variations matches the maximum encountered in Exhibit IV. For values of τ larger than 2.5 seconds, that range, which is already more than substantial, becomes gross. When τ is about 5 seconds, the expected detection time for all-or-nothing suppression ($\eta = 0$) is nearly ten times greater than that with a suppression fraction η of only 0.05 .

For moderate and higher impact rates and moderate single-round periods of suppressive effect, small variations in the suppression fraction thus produce large to gross changes in the expected detection times. As the exhibits show, particularly Exhibit V, fractional suppression strongly limits the increases in expected detection times that can be obtained by increases in the single-round period of suppressive effect; diminished returns from the longer periods are most apparent. Fractional suppression similarly limits the increases in detection times that can be obtained from increases in the rate of impact, and the diminished returns it imposes are equally impressive. Casualty production further limits such increases in expected detection times. The greatest changes occur relative to departures from all-or-nothing suppression; hence, for all but the lowest impact rates, idealizing suppression as a hiatus is ill-advised.

REFERENCES

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